

# Vector Meson Dominance and Rho-Omega Mixing

S. A. Coon\*

*Physics Department, New Mexico State University, Las Cruces, NM 88003*

M. D. Scadron<sup>†</sup>

*Physics Department, University of Arizona, Tucson, AZ 85721*

## Abstract

The scale of a phenomenologically successful charge-symmetry-violating nucleon-nucleon interaction, that attributed to meson exchange with a  $\Delta I = 1$  rho-omega transition, is set by the Coleman-Glashow  $SU(2)$  breaking tadpole mechanism. A single tadpole scale has been obtained from symmetry arguments, electromagnetic meson and baryon measured mass splittings, and the observed isospin violating ( $\Delta I = 1$ ) decay  $\omega \rightarrow \pi^+ \pi^-$ . The hadronic realization of this tadpole mechanism lies in the  $I = 1$   $a_0$  scalar meson. We show that measured hadronic and two-photon widths of the  $a_0$  meson, with the aid of the Vector Meson Dominance model, recover the universal Coleman-Glashow tadpole scale.

PACS numbers: 14.40.cs, 12.70.+q, 13.75.cs, 21.30.+y

[Keyword Abstract] charge symmetry breaking, rho-omega mixing, nucleon-nucleon potential,  $a_0$  scalar meson

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\*Electronic-mail address: coon@nmsu.edu

<sup>†</sup>Electronic-mail address: scadron@physics.arizona.edu

## I. INTRODUCTION

An isospin-violating effective interaction, with the strength of second-order electromagnetic ( $em$ ) theory and labeled  $H_{em}$ , has long been invoked to explain the observed  $\Delta I_z = 1$  meson and baryon diagonal electromagnetic mass splittings, and the observed  $\Delta I = 1$  off-diagonal transitions  $\rho^\circ - \omega$ ,  $\pi^\circ - \eta$ , and  $\pi^\circ - \eta'$ . In particular, it is the effective  $em$   $\rho^\circ - \omega$  transition

$$\langle \rho^\circ | H_{em} | \omega \rangle \approx -4520 \text{ MeV}^2, \quad (1)$$

as found [1] from the observed [2] isospin violating ( $\Delta I = 1$ ) decay  $\omega \rightarrow \pi^+\pi^-$ , which underlies the dominant charge-symmetry-violating (CSV) nucleon-nucleon interaction of refs. [3,4]. The latter CSV NN force is quite successful in explaining the observed charge symmetry violation in nuclear physics. These observations include NN scattering and bound state (the Okamoto-Nolen-Schiffer anomaly) differences in mirror nuclear systems, Coulomb displacement energies of isobaric analog states, isospin-mixing matrix elements relevant to the isospin-forbidden beta decays, and precise measurements of the elastic scattering of polarized neutron from polarized protons [5,6]. In addition, the latter CSV  $\rho^\circ - \omega$  mixing potential is “natural” (*i. e.* dimensionless strength coefficients are  $\mathcal{O}(1)$  in the contact force limit) in the context of low-energy effective Lagrangian approaches to nuclear charge symmetry violation [7]. In spite of the phenomenological success and theoretical plausibility of the CSV potential based upon the effective  $\Delta I = 1$  Hamiltonian density in eq. (1), this potential has been criticized in the recent literature on nuclear charge symmetry violation [8].

Alternative approaches [9] based, not on data and physical Feynman amplitudes, but

upon a “mixed propagator” of field theory, imply a CSV NN potential which is neither consistent with the nuclear data nor with the naturalness criterion. One of the misleading conclusions stemming from a focus on the mixed propagator (only an ingredient of an NN potential) will be discussed in another paper [10]. In this paper we return to the theory behind eq. (1): the Coleman-Glashow tadpole picture [11,12] in which both transitions  $\langle \rho^\circ | H_{em} | \omega \rangle$  and  $\langle \pi^\circ | H_{em} | \eta \rangle$  are given by the tadpole graphs of Fig. 1 and the photon exchange graphs of Fig. 2. We re-examine, in the light of current particle data [13], the numerical accuracy of vector meson dominance (VMD) [14], and then use VMD to link measured decays of the  $I = 1$  scalar meson  $a_0$  to the universality of  $\Delta I = 1$  meson transitions recently established [15]. We close with a discussion of the implications of our results for recent conjectures about a direct  $\omega \rightarrow 2\pi$  coupling [16–20]; *ie.* a decay not based on  $\omega \rightarrow \rho \rightarrow 2\pi$ , which is a  $G$  parity-violating  $\Delta I = 1$  transition.

## II. THE PHOTONIC AND TADPOLE COMPONENTS OF $H_{EM}$

The effective  $\Delta I = 1$  Hamiltonian density  $H_{em}$  in (1) was originally thought [11,12] to be composed of a Coleman-Glashow (CG) nonphotonic contact tadpole part (now couched in the language of the  $u_3$  current quark mass matrix  $\bar{q}\lambda^3 q$ ) along with a photonic part  $H_{JJ}$  involving intermediate photon exchange. This CG tadpole mechanism [11]

$$H_{em} = H_{tad}^3 + H_{JJ} \ , \quad (2)$$

with a *single* tadpole scale, in fact explains the 13 ground state pseudoscalar, vector, octet baryon, and decuplet baryon  $SU(2)$  observed diagonal mass splittings without the introduction of additional free parameters [15].

For the off-diagonal  $\rho^\circ - \omega$  transition, Gatto [21] first showed that the vector meson dominance (VMD) of Fig. 2 predicts the photon exchange contribution

$$\langle \rho^\circ | H_{JJ} | \omega \rangle = (e/g_\rho)(e/g_\omega)m_V^2 \approx 644 \text{ MeV}^2 . \quad (3)$$

In (3) we have used the average  $\rho^\circ - \omega$  mass  $m_V = 776 \text{ MeV}$  along with the updated VMD ratios  $g_\rho/e \approx 16.6$  and  $g_\omega/e \approx 56.3$ , with the latter  $g_\rho$  and  $g_\omega$  couplings found from electron-positron decay rates [13]:

$$\Gamma_{\rho ee} = \frac{\alpha^2}{3} m_\rho (g_\rho^2/4\pi)^{-1} \approx 6.77 \text{ keV} , \quad (4a)$$

$$\Gamma_{\omega ee} = \frac{\alpha^2}{3} m_\omega (g_\omega^2/4\pi)^{-1} \approx 0.60 \text{ keV} , \quad (4b)$$

leading to  $g_\rho \approx 5.03$  and  $g_\omega \approx 17.05$  for  $e = \sqrt{4\pi\alpha} \approx 0.30282$ . Note that Eqs. (4a,4b) imply the ratio  $g_\omega/g_\rho \approx 3.4$ , which is reasonably near the  $SU(3)$  value  $g_\omega/g_\rho = 3$ . Finally, combining the VMD  $H_{JJ}$  prediction (3) with the observed  $H_{em}$  transition in (1), one finds the CG tadpole transition using (2) is

$$\langle \rho^\circ | H_{tad}^3 | \omega \rangle \approx -4520 \text{ MeV}^2 - 644 \text{ MeV}^2 \approx -5164 \text{ MeV}^2 . \quad (5)$$

In fact this off-diagonal CG tadpole scale of (5) extracted from  $\omega \rightarrow 2\pi$  data combined with the VMD scale of (3) is quite close to the CG tadpole scale predicted from the  $SU(3)$  diagonal vector meson mass splittings [22]. If the  $\omega$  is assumed to be pure nonstrange, the  $SU(3)$  prediction becomes

$$\langle \rho^\circ | H_{tad}^3 | \omega \rangle = \Delta m_{K^*}^2 - \Delta m_\rho^2 \approx -5120 \text{ MeV}^2 , \quad (6)$$

obtained using the 1996 PDG values,  $m_{K^{*+}} \approx 891.6$  MeV,  $m_{K^{*0}} \approx 896.1$  MeV so that  $\Delta m_{K^*}^2 = m_{K^{*+}}^2 - m_{K^{*0}}^2 \approx -8040$  MeV<sup>2</sup>. While  $m_{\rho^+} \approx 766.9$  MeV, the more elusive  $\rho^0$  mass at [23]  $m_{\rho^0} \approx 768.8$  MeV then requires  $\Delta m_\rho^2 = m_{\rho^+}^2 - m_{\rho^0}^2 \approx -2920$  MeV<sup>2</sup>. The difference between  $\Delta m_{K^*}^2$  and  $\Delta m_\rho^2$  above then leads to the right hand side of (6).

Since only a slight change of  $m_{\rho^0}$  above can shift  $\Delta m_{K^*}^2 - \Delta m_\rho^2$  by more than 10%, it is perhaps more reliable to exploit  $SU(6)$  symmetry between the pseudoscalar and vector meson masses,  $m_{K^*}^2 - m_\rho^2 = m_K^2 - m_\pi^2$ . But because this  $SU(6)$  relation is valid to within 5%, it is reasonable to assume the  $SU(6)$  mass difference  $\Delta m_{K^*}^2 - \Delta m_\rho^2 = \Delta m_K^2 - \Delta m_\pi^2$  also holds. Then the  $\rho^0 - \omega$  tadpole transition (6) is predicted to be [15]

$$\langle \rho^0 | H_{tad}^3 | \omega \rangle = \Delta m_{K^*}^2 - \Delta m_\rho^2 = \Delta m_K^2 - \Delta m_\pi^2 \approx -5220 \text{ MeV}^2, \quad (7)$$

because pseudoscalar meson data [13] requires  $\Delta m_K^2 = m_{K^+}^2 - m_{K^0}^2 \approx -3960$  MeV<sup>2</sup> and  $\Delta m_\pi^2 = m_{\pi^+}^2 - m_{\pi^0}^2 \approx -1260$  MeV<sup>2</sup>, leading to the right hand side of (7). Comparing the similar tadpole scales of  $\sim -5200$  MeV<sup>2</sup> in (5),(6), and (7), we might deduce from this consistent picture that  $\langle \rho^0 | H_{em} | \omega \rangle$  in turn is predicted to have the scale  $\langle \rho^0 | H_{em} | \omega \rangle \approx -4500$  MeV<sup>2</sup>, as was found from the Barkov  $\omega \rightarrow 2\pi$  data [1,2].

To emphasize that the above tadpole scale (5)-(7) of the off-diagonal  $\Delta I = 1$   $\rho^0 - \omega$  transition also holds for the diagonal electromagnetic mass differences as well, we briefly review the well-measured pseudoscalar  $\pi$  and  $K$   $em$  mass splittings. It has long been known [24] that  $\Delta m_\pi^2$  is essentially due to the photonic self-interaction mass shifts of the charged and uncharged pions [25]. As noted in Ref. [15], this familiar idea takes the form in the tadpole picture ( $(H_{em})_{\pi^+} = \langle \pi^+ | H_{em} | \pi^+ \rangle$ , etc):

$$\Delta m_\pi^2 \equiv (H_{em})_{\Delta\pi} \equiv (H_{em})_{\pi^+} - (H_{em})_{\pi^0} = (H_{tad}^3)_{\Delta\pi} + (H_{JJ})_{\Delta\pi} = (H_{JJ})_{\Delta\pi} , \quad (8a)$$

where the first equality is due to the CG decomposition (2) and the second equality is because  $(H_{tad}^3)_{\Delta\pi} = 0$  due to  $SU(2)$  symmetry. However  $(H_{tad}^3)_{\Delta K}$  does not vanish in the analogous CG *kaon* mass splitting relation

$$\Delta m_K^2 \equiv (H_{tad}^3)_{\Delta K} + (H_{JJ})_{\Delta K} . \quad (8b)$$

Then subtracting (8a) from (8b) while using the Dashen PCAC observation [26]

$$(H_{JJ})_{\pi^0} = (H_{JJ})_{K^0} = (H_{JJ})_{\bar{K}^0} = 0 , \quad (H_{JJ})_{\pi^+} = (H_{JJ})_{K^+} , \quad (8c)$$

which is strictly valid in the chiral limit, one is led [15] to the diagonal pseudoscalar meson tadpole scale

$$(H_{tad}^3)_{\Delta K} \equiv (H_{tad}^3)_{K^+} - (H_{tad}^3)_{K^0} = \Delta m_K^2 - \Delta m_\pi^2 \approx -5220 \text{ MeV}^2 . \quad (9)$$

Extending  $\Delta m_K^2 - \Delta m_\pi^2$  to  $\Delta m_{K^*}^2 - \Delta m_\rho^2$  via  $SU(6)$  along with  $(H_{tad}^3)_{\Delta K} = (H_{tad}^3)_{\Delta K^*}$  also by  $SU(6)$  symmetry yields the diagonal vector meson tadpole scale

$$(H_{tad}^3)_{\Delta K^*} \equiv (H_{tad}^3)_{K^{*+}} - (H_{tad}^3)_{K^{*0}} = \Delta m_{K^*}^2 - \Delta m_\rho^2 \approx -5120 \text{ MeV}^2 . \quad (10)$$

Thus we see that the off-diagonal  $\rho^0 - \omega$  tadpole scales (5) - (7) together with the diagonal tadpole scales in (9) and (10) are indeed universal. This -5200 MeV<sup>2</sup> scale also is approximately valid for diagonal baryon masses when one uses quadratic mass formulae for baryons [15].

### III. TADPOLE MECHANISM AND THE $a_0(980)$

The  $I = 1$   $a_0$  scalar meson is assumed to play a unique role in the Coleman-Glashow  $\Delta I = 1$  tadpole mechanism which describes  $SU(2)$  mass differences and mixing among hadrons [11,12,27]. Furthermore this meson is also almost unique among the scalar mesons in that it is experimentally well established with known decay parameters. Therefore one can test the universal tadpole scale of the previous section against experimental data from an entirely different sector. We shall see that this confrontation yields yet another consistent pattern of a  $\Delta I = 1$  universal tadpole scale.

More specifically the  $\Delta I = 1$   $em$  tadpole graphs of Figs. 1 are controlled by the  $I = 1$   $a_0(980)$  pole for both  $a_0 \rightarrow \omega\rho^0$  and  $a_0 \rightarrow \eta\pi^0$  transitions. The unknown tadpole  $\langle 0|H_{tad}^3|a_0\rangle$  and the  $a_0$  propagator cancel out of the ratio

$$\frac{\tilde{F}_{a_0\rho^0\omega}}{F_{a_0\pi^0\eta_{NS}}} \approx \frac{\langle \rho^0|H_{tad}^3|\omega\rangle}{\langle \pi^0|H_{tad}^3|\eta_{NS}\rangle} , \quad (11)$$

where  $\tilde{F}_{a_0\rho^0\omega} \equiv m_{a_0}^2 F_{a_0\rho^0\omega}$ . The left hand side of (11) can be related to the experimental ratio obtained from the PDG rate  $\Gamma_{a_0\gamma\gamma} = (0.24 \pm 0.08)$  keV (assuming  $\Gamma_{a_0\pi\eta}$  is overwhelmingly dominant) divided by  $\Gamma_{a_0\pi\eta} \approx 75$  MeV, midway between the PDG range (50-100) MeV:

$$r = \frac{\Gamma_{a_0\gamma\gamma}}{\Gamma_{a_0\pi\eta}} \approx \frac{0.24 \text{ keV}}{75 \text{ MeV}} \approx 3.2 \times 10^{-6} . \quad (12)$$

This relation is again the vector meson dominance (VMD) model turning the vector mesons of Fig 2 and (11) into the  $\gamma$ 's of (12), since the “rate” for  $a_0 \rightarrow \rho^0\omega$  cannot be directly measured.

To illustrate the VMD model [14] in this context, we first study  $\omega\pi\gamma$  coupling (times the

usual Levi-Civita factor  $\epsilon_{\mu\nu\alpha\beta}k'^\mu k^\nu \epsilon^\alpha \epsilon^\beta$ ) by comparing it to  $\pi^0\gamma\gamma$  coupling (divided by 2 due to Bose symmetry)

$$F_{\omega\pi^0\gamma} = (g_\omega/e)F_{\pi^0\gamma\gamma}/2 \approx 0.704 \text{ GeV}^{-1} \quad (13a)$$

by virtue of VMD turning an  $\omega$  into a  $\gamma$ . Here  $F_{\pi^0\gamma\gamma} = \alpha/(\pi f_\pi) \approx 0.025 \text{ GeV}^{-1}$  as found from the axial anomaly or using instead the  $\pi^0\gamma\gamma$  rate of 7.6 eV [13]. This VMD prediction (13a) is in excellent agreement with the measured value [13]

$$F_{\omega\pi^0\gamma} = \sqrt{12\pi\Gamma_{\omega\pi^0\gamma}/k^3} \approx 0.704 \text{ GeV}^{-1} , \quad (13b)$$

where the amplitude  $F_{\omega\pi^0\gamma}$  is also weighted by  $\epsilon_{\mu\nu\alpha\beta}k'^\mu k^\nu \epsilon^\alpha \epsilon^\beta$ . A similar VMD prediction for  $\rho \rightarrow \pi^0\gamma$  is also quite good: The VMD amplitude is

$$F_{\rho\pi^0\gamma} = (g_\rho/e)F_{\pi^0\gamma\gamma}/2 \approx 0.208 \text{ GeV}^{-1} \quad (13c)$$

using  $g_\rho \approx 5.03$  found from (4a), while data implies

$$F_{\rho\pi^0\gamma} = \sqrt{12\pi\Gamma_{\rho\pi^0\gamma}/k^3} \approx 0.222 \text{ GeV}^{-1} , \quad (13d)$$

as extracted from the PDG tables in [13].

Given this justification of VMD in equs. (13), we follow Bramon and Narison [28] and use VMD to link the CG tadpole mechanism of Figs. 2 with the observed properties of the  $a_0$  meson. We return to (11) and note that the  $a_0\rho^0\omega$  coupling in (11) is divided by 2 (as it was in (13a)) when applying VMD to the identical photon transition:

$$F_{a_0\rho^0\omega} \approx (g_\omega/e)(g_\rho/e)F_{\pi^0\gamma\gamma}/2 . \quad (14)$$



Since both  $F_{a_0\rho^0\omega}$  and  $F_{a_0\gamma\gamma}$  are weighted by the covariant form  $\epsilon_\mu^* \epsilon_\nu^*(k' \cdot k g^{\mu\nu} - k^\mu k'^\nu)$  which squares up to  $(k' \cdot k)^2 = 2(m_{a_0}^2/2)^2$ , the desired amplitude  $\tilde{F}_{a_0\rho^0\omega} = m_{a_0}^2 F_{a_0\rho^0\omega}$  has the same  $(\text{GeV})^1$  mass dimension as does  $F_{a_0\pi^0\eta_{NS}}$ . The latter is given as

$$F_{a_0\pi^0\eta_{NS}} = F_{a_0\pi^0\eta} / \cos \phi \approx 1.35 F_{a_0\pi^0\eta} \quad (15)$$

for the  $\eta' - \eta$  mixing angle  $\phi \approx 42^\circ$  in the  $NS - S$  quark basis [29].

On the other hand, the theoretical branching ratio rates become with VMD [28]

$$r = \frac{\Gamma_{a_0\gamma\gamma}}{\Gamma_{a_0\pi\eta}} = \frac{1}{4} \left| \frac{k_\gamma}{k_\eta} \right| \frac{\tilde{F}_{a_0\gamma\gamma}^2}{F_{a_0\pi\eta}^2} = \frac{4}{4} \left| \frac{k_\gamma}{k_\eta} \right| \left( \frac{e}{g_\omega} \right)^2 \left( \frac{e}{g_\rho} \right)^2 \frac{\tilde{F}_{a_0\rho^0\omega}^2}{F_{a_0\pi\eta}^2} , \quad (16)$$

where  $k_\gamma = 492$  MeV,  $k_\eta = 321$  MeV, so that  $|k_\gamma/k_\eta| \approx 1.53$ . The factor of  $\frac{1}{4}$  in (16) corresponds to Feynman's rule of  $\frac{1}{2}$  for two identical final-state photons, times the numerator factor of  $\frac{1}{2}$  in (16) coming from the square of the covariant factor  $\epsilon_\mu^* \epsilon_\nu^*(k' \cdot k g^{\mu\nu} - k^\mu k'^\nu)$  (times  $m_{a_0}^2$  which is absorbed into the definition of  $\tilde{F}_{a_0\gamma\gamma}$ ). Finally a factor of 4 in the numerator of the right hand side of (16) is due to the square of the VMD relation (14).

Substituting the observed  $r$  from (12) back into the theoretical ratio (16) and converting to the  $\eta_{NS}$  basis using (15) then leads to the amplitude ratio

$$\frac{\tilde{F}_{a_0\rho^0\omega}}{F_{a_0\pi^0\eta_{NS}}} \approx 1.0 \quad , \quad (17a)$$

a result which stems only from observed properties of the  $a_0$  meson and VMD.

If one goes further and identifies the transition  $\langle 0 | H_{em} | a_0 \rangle$  as the Coleman-Glashow tadpole, the VMD-phenomenological estimate of unity for the ratio (17a) requires the tadpole ratio in (11) and in Figs. 1 also to be unity

$$\frac{\langle \rho^0 | H_{tad}^3 | \omega \rangle}{\langle \pi^0 | H_{tad}^3 | \eta_{NS} \rangle} \approx 1.0 \quad , \quad (17b)$$

This hadronic picture of the CG tadpole is consistent with the universal  $SU(6)$  tadpole scale already obtained in Ref. [15] and reviewed in Section 2.

Implicit in this identification is the conventional  $\bar{q}q$  assignment of the  $a_0$ . According to ref. [28], the tadpole mechanism fails to predict the experimental  $\Gamma_{a_0\gamma\gamma}$  if the  $a_0$  is considered to be a  $\bar{q}qq\bar{q}$  state. Recent  $K$ -matrix analyses of meson partial waves from a variety of three-meson final states obtained from  $\pi^-p$  and  $p\bar{p}$  reactions show rather convincingly that both the  $I = 1$   $a_0(980)$  and  $I = 0$   $f_0(980)$  mesons are formed from the bare states which are members of the lowest  $\bar{q}q$  nonet [30]. Recent theoretical developments supporting the assignment of the  $a_0$  to the scalar meson  $\bar{q}q$  nonet are reviewed in Ref. [31].

The modern identification of the tadpole scale with the mass difference of the up and down current quarks:

$$H_{tad}^3 = \frac{1}{2}(m_u - m_d)\bar{q}\lambda_3q, \quad (18)$$

suggests a parallel treatment of the hadronic and two-photon couplings of the  $a_0$  based on the three point function method in QCD sum rules. One begins with VMD to express another measured ratio

$$\frac{\Gamma_{a_0\gamma\gamma}}{\Gamma_{\pi^0\gamma\gamma}} \approx \frac{m_{a_0}^3}{m_\pi^3} \frac{g_{a_0\rho^0\omega}}{g_{\pi^0\rho^0\omega}} \quad (19)$$

in terms of the strong coupling constants  $g_{a_0\rho^0\omega}$  and  $g_{\pi^0\rho^0\omega}$ . The latter coupling constant ratio is then estimated with the aid of QCD sum rules which bring in the up and down current quark masses. The result is again a reasonable value for  $\Gamma_{a_0\gamma\gamma}$  and the ratio  $r$  of (12). This QCD sum rule treatment of the  $a_0$  decays is consistent with, but does not really

give new information about the Coleman-Glashow tadpole. So we do not describe it further and refer to Refs. [28,32] for a detailed account of this QCD sum rule program.

#### IV. DISCUSSION

In Section 3, vector meson dominance was used to link measured decays of the  $I = 1$  scalar meson  $a_0$  to the universality of  $\Delta I = 1$  meson transitions  $\langle \rho^\circ | H_{em} | \omega \rangle$  and  $\langle \pi^\circ | H_{em} | \eta \rangle$  recently established [15]. One element of this universality is the value of the effective  $em$   $\rho^\circ - \omega$  transition found [1] from the observed [2] isospin violating ( $\Delta I = 1$ ) decay  $\omega \rightarrow \pi^+ \pi^-$ . To obtain this value, the decay is analyzed as  $\omega \rightarrow \rho \rightarrow 2\pi$  [4]. It has recently been asserted [16] that a direct  $\omega \rightarrow 2\pi$  coupling is not only necessary on general principles, but that a significant coupling is supported by a theoretical QCD sum rule analysis of a isospin-breaking correlator of vector currents. This assertion has prompted reanalyses of the data on  $e^+ e^- \rightarrow \pi^+ \pi^-$  [17,18], and the modeling of this putative contact  $\omega \rightarrow 2\pi$  term in two quark based models of  $\rho^\circ - \omega$  mixing [19,20]. The results of these three investigations are somewhat mixed. Here they are given as the ratio  $G = g_{\omega_I \pi \pi} / g_{\rho_I \pi \pi}$ , where  $\omega_I$  and  $\rho_I$  are the basis states of pure isospin  $I = 0$  and  $I = 1$ , respectively. The data analysis of Ref. [18] suggests  $G \approx 0.10$ , the coupled Dyson-Schwinger equations approach [20] finds a value five times smaller ( $G \approx 0.017$ ), and the generalized Nambu-Jona-Lasinio model [19] predicts a value which is a further factor of four smaller ( $G \approx 0.004$ ). The last very small ratio would make direct  $\omega$  decay a relatively unimportant contribution to the calculation of  $\rho^\circ - \omega$  mixing from the data. On the other hand, the isospin breaking from direct  $\omega$  decay suggested by the data analysis [18] is a huge 10% rather than the few percent usually found for isospin breaking

(cf. the 2% Coleman-Glashow ratio reviewed in the Appendix of Ref. [15]). This in turn drives the value of  $\langle \rho^0 | H_{em} | \omega \rangle$  up to  $\approx -6830 \text{ MeV}^2$ , rather far from the value of  $\approx -4520 \text{ MeV}^2$  (quoted in Eq. (1)) obtained from the same data when this putative contact term is ignored. It is the latter value which was shown in sections 2 and 3 to be consistent with the universality discussed there. The most recent extraction of a  $\rho^0 - \omega$  mixing parameter from the data eschews such a separation into a contact  $\omega \rightarrow \pi^+ \pi^-$  and mixing  $\omega \rightarrow \rho \rightarrow 2\pi$  term on the grounds that it is model dependent [33,34]. As we have seen, a significant contact G-parity violating  $\omega \rightarrow 2\pi$  coupling would increase in magnitude the value of  $\langle \rho^0 | H_{em} | \omega \rangle$  to such an extent that it would not be consistent with the off-diagonal  $\rho^0 - \omega$  tadpole scales (6) and (7), nor with the diagonal tadpole scales in (9) and (10), nor with the diagonal tadpole scales obtained from the baryon mass differences [15]. Furthermore, such a large value of  $\langle \rho^0 | H_{em} | \omega \rangle$  is not consistent with the  $\Delta I = 1$  universal tadpole scale obtained in (17) with the aid of the Vector Meson Dominance (VMD) model. In view of the inconsistency with the global Coleman-Glashow picture and the limited support from quark based models [19,20] for a contact  $\omega \rightarrow 2\pi$  which violates G-parity, it is instructive to look at the analogue four-point contact term in the *strong* decay  $\omega \rightarrow 3\pi$ . This term was introduced on general grounds some time after the suggestion [35] that  $\omega \rightarrow \rho\pi \rightarrow 3\pi$  will dominate the  $\omega \rightarrow 3\pi$  transition. This dominance of this VMD  $\omega\rho\pi$  pole diagram model has been confirmed by the experimental study of the  $e^+e^- \rightarrow 3\pi$  reaction [36]. In fact, a contact term large enough to satisfy a low-energy theorem [38] in the pseudoscalar sector (the AVV anomaly) spoils agreement with this data. The history of the fate of the contact  $\omega \rightarrow 3\pi$  term can be traced in Ref. [37], which concludes that “nowadays the existence and magnitude of the contact term can be

extracted neither from theory, nor experiment.”

We suggest that a similar fate may be store for the proposed G-parity violating  $\omega \rightarrow 2\pi$  contact term. While its effects cannot be cleanly isolated from data [17,18], in contrast to the proposed strong interaction  $\omega \rightarrow 3\pi$  contact term, nevertheless the introduction of this G-parity violating  $\omega \rightarrow 2\pi$  contact term is inconsistent with the  $SU(6)$  prediction (7), the universal CG tadpole scale [15], and, as shown in section 3, the measured decay properties of the  $a_0$  meson.

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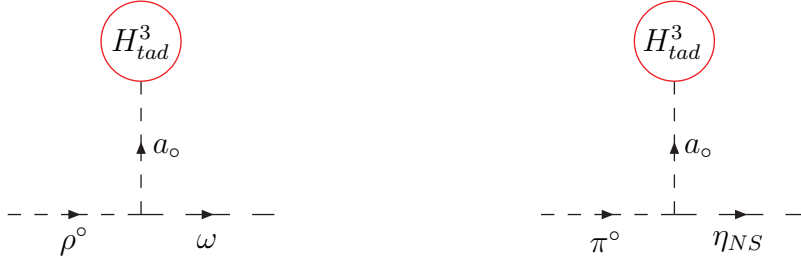


Figure 1  $a_0$  meson tadpole diagrams for the CSB  $\Delta I = 1$  transitions  $\langle \rho^\circ | H_{tad}^3 | \omega \rangle$  and  $\langle \pi^\circ | H_{tad}^3 | \eta_{NS} \rangle$ . According to Coleman and Glashow [11], these are diagrams that can be broken into two parts, connected only by the scalar meson  $a_0$  line, such that one part is the scalar tadpole  $\langle 0 | H_{tad}^3 | a_0 \rangle$ , represented by the circle, and the other part involves only the  $SU(3)$  invariant strong interactions. The latter interactions, in this case,  $a_0 \rightarrow \omega \rho^0$  and  $a_0 \rightarrow \eta \pi^0$  transitions, are represented by the coupling constants of (11).



Figure 2 The current-current contribution  $\langle \rho^\circ | H_{JJ} | \omega \rangle$  to  $\langle \rho^\circ | H_{em} | \omega \rangle$ .